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TOWARD A STABLE DIAGNOSTIC REPRESENTATION OF STUDENTS' ERRORS IN ALGEBRA

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Toward a Stable Diagnostic Representation of Students' Errors in Algebra

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Running head: Stability of Error Models

Abstract

Diagnoses of students' performance on procedural mathematical tasks need to display a certain level of stability and robustness if they are to be used as the basis for remediation, particularly with computer-delivered instruction. The purpose of this study was to compare two diagnostic approaches for describing students' errors in algebra - - a bug analysis and a rule-space analysis -- with the goal of investigating the relative stability of the diagnoses derived from these approaches. Consistent with the findings of recent studies, a relatively large number of bugs were unstable; stable bugs tended to be infrequent. In contrast, the results of the rule-space analysis yielded relatively more stable diagnoses. The results were discussed in light of their consequences for designing remediation.

Toward a Stable Diagnostic Representation of Students' Errors in Algebra

Cognitive scientists have proposed and investigated several computational mechanisms for explaining students' procedural errors in mathematics, including Repair theory (Brown & Burton 1978; Brown & VanLehn, 1980; VanLehn, 1990), misgeneralization (Sleeman, 1984a, 1984b), deletion (Young & O'Shea, 1981), and the competing-rules model (Payne & Squibb, 1990). Regardless of the adequacy of the proposed mechanism for accounting for how errors are generated (whether in response to an impasse or as the result of misgeneralizing a learned rule), a persistent concern about existing models of errors is their instability (VanLehn, 1982; Sleeman, Kelly, Martinak, Ward & Moore, 1989; Payne & Squibb, 1989).

In order to investigate the stability of the diagnoses produced by mal-rules, researchers have observed the recurrence of mal-rules within a test (Payne & Squibb, 1990; Blando, Kelly, Schneider & Sleeman, 1989; Tatsuoka, Birenbaum & Arnold, 1989) or across tests (Payne & Squibb, 1990; Sleeman, Kelly, Martinak, Word & Moore, 1989; VanLehn, 1982; Bricken, 1987). Both within and across testings, a large number of mal-rules have been found to be unstable, and the stable ones tend to be very infrequent. Consequently, doubts have arisen regarding the potential usefulness of mal-rules for remedial purposes (Sleeman, et al., 1989).

The kernel of the problem posed by unstable mal-rules as cognitive models of error was articulated by VanLehn (1982, p. 46): "[Lack of stability] challenges us to change our image of a bug as something that necessarily exists over time as part of the child's long term beliefs. . ." In other words, for the purposes of remediation we cannot be confident that a buggy analysis of a student's performance in a mathematics task necessarily produces a stable student model. In order for human or machine-delivered remediation to proceed on a reliable basis, a stable diagnosis is a necessary, if not sufficient, prerequisite.

An alternative approach to error diagnosis is to refocus attention to the source of the impasse that causes buggy behavior (stable or unstable) on the part of the student, rather than attempting to model the cognitive response to the impasse. For example, a number of mal-rules have been identified when students are confronted with an equation in the form $ax = b$, including $x = b$ (Sleeman et al., 1989), $x = b - a$ (Sleeman et al., 1989; Payne & Squibb, 1990), $x = -(a + b)$ (Gutvirtz, 1989), $x = a - b$ (Gutvirtz, 1989), and $x = a + b$ (Gutvirtz, 1989; Payne & Squibb, 1990). What each of these bugs has in common is that each is a response to the students' nonmastery of the subskill of dividing across by the coefficient of x. The cause of the impasse is the nonmastered subskill.

As noted by VanLehn (1982), it is extremely difficult to tease out of a set of items the presence or absence of subskills using the pattern of right and wrong answers. The rule space technique, developed by Tatsuoka, was designed to handle this problem (e.g., Tatsuoka, 1983, 1985, 1990, 1991; Tatsuoka & Tatsuoka, 1987). The rule-space classifies students into knowledge states that consist of response patterns that are described in terms of mastery or nonmastery of predetermined task attributes. The analysis collapses across items, and classifies students according to factors (subskills in this case) that are identified to be integral to the successful completion of an item or subsets of items. In this paper we report on the results of a rule space analysis of students' performance on linear equations in one unknown in which the "attributes" were described at the level of the source of the student's errors (e.g., "has not mastered the distributive law").

More technically, rule-space is a probabilistic approach whose purpose is to identify the examinee's state of knowledge, based on an analysis of the task's cognitive requirements. The following is a brief presentation of the rule-space approach:

First the task's cognitive requirements (also called attributes) are specified. From these, an item x attribute incidence matrix, Q, is constructed. This matrix is binary and of order K x m (the number of attributes x the number of items). If q_{kj} is the (k,j) element of this matrix (where k indicates an attribute and j indicates an item) then, $q_{kj}=1$ if item j involves attribute k, and $q_{kj}=0$ otherwise. Concepts represented by unobservable variables that can be derived from the incidence matrix Q are called cognitive states (or attribute patterns). Boolean Description Functions are used systematically to determine those cognitive states and map them into observable item-score patterns (called ideal item-score patterns) (see Tatsuoka, 1991; Varadi & Tatsuoka, 1989). Once the ideal item-score patterns are obtained, the actual data are considered.

The rule space then maps the actual item-score patterns of the examinees onto the cognitive states in order to find the ideal item-score pattern closest to a given student's actual response pattern. This pattern classification problem is handled by the rule-space model. Item Response Theory (IRT) is utilized for formulating the classification space, which is a Cartesian product space of IRT ability/proficiency, θ, and variable(s), ζ, which measure the unusualness of item-score patterns (Tatsuoka, 1984; Tatsuoka & Linn, 1983). Bayes' decision rules are used for the classification of an examinee into the cognitive states. Once this classification has been carried out, one can indicate which attributes a given examinee is likely to have mastered or failed to master.

The present study examined the stability of the diagnostic models produced by rule space and those produced by a bug analysis. Rule space and buggy analyses were applied to two sets of algebra items that were designed to be parallel in terms of their attributes (task requirements).

Methodology

Subjects

The sample consisted of 231 8th and 9th graders (ages 14-15) from an integrated junior high school in Tel Aviv. Fifty-seven percent of the subjects were girls. The students studied mathematics in high and low achievement groupings (106 in the former and 125 in the latter).

Instruments and procedures

A 32-item diagnostic test in linear algebraic equations in one unknown was developed by Gutvirtz (1989) based on a detailed task analysis including a procedural network and a mapping sentence (e.g. Birenbaum & Shaw, 1985). The test was developed for the purpose of identifying students' bugs in solving those equations. All items were open-ended and the students were asked to show all solution steps. The present study used a subset of those items which consisted of two sets of nine parallel items attribute-wise: in set 1 (items 1, 2, 3, 6, 8, 10, 11, 12, 13); in set 2 (items 25, 24, 27, 23, 18, 19, 20, 22, 30). (The 18 items appear in Appendix A).

The correlation coefficient between the scores on the two sets was 0.85. The item difficulty indices (percent correct) in set 1 (items 1, 2, 3, 6, 8, 10, 11, 12, 13) ranged from 0.63 to 0.93 with an average of 0.78. In set 2 (items 25, 24, 27, 23, 18, 19, 20, 22, 30) the range was from 0.53 to 0.91 with an average of 0.76. The item discrimination indices (item-total correlations) in set 1 ranged from 0.49 to 0.75, with an average of 0.61. In set 2 the range was from 0.51 to 0.73, with an average of 0.61. The correlation coefficients between the two sets with respect to item difficulties and item discrimination indices were 0.93 and 0.82, respectively.

The bug analysis:

On the basis of a detailed examination of the procedures followed by the students in solving the test items, 34 mal-rules (bugs) were identified (see Gutvirtz, 1989 for a listing of the bugs). A bug X item matrix was then constructed. The entries of this matrix were the answers to the test items produced by applying the mal-rules. The students' actual answers were then matched to the entries in the bug matrix and coded accordingly. Of the actual responses, 94.6% were matched to identified bugs or to the correct rule, the rest were either unidentified bugs or clerical errors. Of the 231 subjects, 50 answered all 18 items correctly, and were therefore excluded from subsequent analysis. The coded responses included 38 different codes: one indicating the correct answer, one indicating unidentified errors, one indicating clerical errors, one indicating omissions, and the rest indicating the various identified bugs. The codes for parallel items were then compared.

Matches and mismatches were counted across the nine pairs of parallel items for each of the 181 examinees, and classified according to the following primary categories: (a) matched correct (1,1); (b) one correct and one error (1,0; 0,1); (c) matched bug; and (d) nonmatched errors (nonmatched bugs or unidentified errors).

The rule-space analysis:

1. Determining the attributes: A set of 11 attributes was specified for a solution strategy for solving the items (see Table 1) and used to produce an incidence matrix (see Appendix A). For example, the following attributes are appropriate for item 10 (note that "evaluating" means that the student decides from the outset not to rewrite the equation in standard from until the final step -- thereby avoiding a negative x-term):

$4(2x + 3) = 10x$	("evaluating" the equation and applying the distributive law)
$8x + 12 = 10x$	(subtracting a term from both sides)
$12 = 10x - 8x$	(adding or subtracting variable terms)
$12 = 2x$	(dividing across by the coefficient of x, when a<b)
$6 = x$	(applying the symmetry law)
$x = 6$	

See the operations denoted for item 10 in Appendix A, and the attribute list in Table 1.

Insert Table 1 about here

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2. Testing the adequacy of the attribute matrix: A multiple regression with item difficulties as the dependent variable and the 11 attribute vectors of the Q matrix as the independent variables was performed. The set of attributes accounted for 94% of the variance (R^2=.94; R^2_{adj}=.89).

3. The BILOG program (Mislevy & Bock, 1983) was used for estimating the item parameters (a's and b's) of the IRT two-parameter logistic model. The b values for the first subtest correlated 0.90 with the b values for the second subtest. The correlation for the a values of the two subsets was 0.75. The b values of the first and second subtests ranged

from -2.12 to -.26 and from -1.90 to .04, respectively. The a values of the first and second subtests ranged from .68 to 1.52 and from .72 to 1.55, respectively.

4. The BUGLIB program (Varadi & Tatsuoka, 1989) was used for deriving the ideal score patterns corresponding to the attribute mastery patterns that constituted the groups into which the students' actual response patterns were classified. As a result, 78 groups (knowledge states) were generated. The same program was also used for the classification. The classification was applied to each subset of items separately; that is, each student was classified twice, once according to his or her responses to set 1, and once according to the responses to the parallel set, set 2.

5. The results of the classifications (i.e., the students' attributes patterns on the two sets of 11 attributes) were then compared. Of the 231 subjects, 50 answered all 18 items correctly, and 4 answered all items incorrectly; thus 54 subjects were therefore excluded from subsequent analysis. Matches and mismatches were counted across the 11 pairs of attributes for each of the 177 examinees and classified according to the following primary categories: (a) matched mastery (1,1); (b) mastery/nonmastery (1,0; 0,1); and (c) matched nonmastery (0,0).

Results

Mal-rule stability

Before presenting the results at the group level, two examples of the bug analysis for the two parallel sets of items for two students are presented in Table 2. A comparison of the two row-vectors for the first student (No. 13) indicated that he consistently answered correctly one pair of parallel items and consistently applied incorrect rules on five pairs of items. On the remaining two pairs of items he inconsistently applied different mal-rules, and on one pair he omitted the response to one item. Thus the percentage of matched correct responses for this student was 11.11%, the percentage of matched bugs was 55.56%, the percentage of non-matched errors was 33.33%.

The second student (No. 82) also correctly answered one pair of parallel items (11.11%), she consistently applied the same bug to four pairs (44.44%), and the percentage of unmatched errors was 44.44%. In no case did either of the students get one of the items in a pair correct and the other item incorrect.

Insert Table 2 about here

It should be noted that although the two students had the same pattern of correct/incorrect answers, their bugs differed in type and frequency. While the first student was consistently applying three mal-rules [A: a + x => ax; B: ax + a => (a + a) x; and C: ax = b => x = a/b (when a > b)], the second student consistently applied only one mal-rule [F: ax • b = c => ax = c @ b; when • is "+" then @ is "-" and vice versa].

Evaluated at the group level, 64.58% of the total matched responses across the 9 pairs of items were matched correct answers. A further 18.97% included one correct and one incorrect response, and 6.38% were nonmatched errors (including nonmatched bugs and unidentified errors). The remaining 10.07% of the total matched responses were matched bugs. To better understand this final percentage, note that for the right/wrong scoring the overall match of correct (1,1) and incorrect (0,0) responses was 81.03%, (64.58% matched correct and 16.45% matched incorrect). Thus, of the incorrect pairs (0,0), 61% consisted of matched bugs. Greater insight into the percentage of matched bugs may be gained by inspecting Table 3. This table presents the frequency of stable bugs for each pair of parallel items. As can be seen, the thirty-four stable bugs are sparsely distributed across the nine item-pairs.

Insert Table 3 about here

Attribute stability.

Before presenting the results at the group level, the following is an example of the rule-space analysis at the individual level. The example is based on the responses given by the two students whose bug analyses were presented above. Since both answered correctly the same pair of items, (No. 4 in each subset) and erred on all the other items, their attribute mastery pattern is identical. The two vectors of 11 attributes for these students, as derived from their responses to the two parallel subsets, are presented in Table 4. A comparison of the two row-vectors indicates that they are identical; i.e., they reflect the same knowledge state. Thus, for both students, the percentage of matched mastery attributes (1,1) is 18.18%, the percentage of matched nonmastery is 81.82% and that of one mastery and one nonmastery is 0.00%. The students' response pattern to the test items perfectly matched the knowledge state indicating mastery of only two attributes (9 and 11, see Table 1), and nonmastery of all the rest.

Insert Table 4 about here

At the group level the percentage of matched and nonmatched responses across the 11 pairs of attributes are as follows: 80.18% of the responses yielded a match [63.38% of the responses for mastery and 16.80% for nonmastery (0,0)]. The percentage of nonmatched attributes [mastery/non mastery or (1,0), (0,1) patterns] was 19.82%. The correlation coefficient between the mastery scores derived from the two subsets in the total sample, which is an index of the reliability of these scores, was 0.79. Note that at the item level (0/1 scores) that coefficient was 0.85. The percentage of mastery for each attribute may be found in Appendix A.

Discussion

The results of the present study showed that a rule space analysis of attributes defined in terms of the subskill components of a procedural task produced a relatively stable within-test student model. On the bug-level, although our analysis found more stable bugs than were previously reported during a single testing session (see data on School 3 in Payne and Squibb, 1989), many bugs had very low frequencies. While an umastered skill is likely to remain unmastered (without intervening tutoring), the impasse that results from it may trigger many buggy responses (some stable and infrequent, and many unstable). For the same reason, a measure of mastery/nonmastery of a subskill is likely to demonstrate stabiilicy across testings (and be more stable than a corresponding buggy analysis), but this prediction needs to be tested empirically.

Advantages of Attribute Analyses over Bug Analyses

1. A clear advantage of focusing on the deficient subskills (as attributes) is that they are known mathematical entities. Consequently, remedial prescriptions for the teacher are in terms that are immediately meaningful for them (see Putnam, 1987). Bugs, on the other hand, are often a mystery both to the researcher and the teacher because, "many bugs have conditions and actions that simply do not appear in any arithmetic algorithm . . . " (VanLehn, 1990, p. 6, original emphasis).

2. The identified attributes are integral subcomponents of the task; thus if a student fails the task, the failure, at least at the procedural level, must be traceable to one or more deficiencies in these subskills (if the subskill analysis was exhaustive). The generative nature of bugs, on the other hand, means that a given catalog of bugs may explain errors for the data reported in one study, but not in another and, within the same study, bugs applicable in one school may not be applicable in a different school (Payne & Squibb, 1989). The capriciousness of bugs can lead to inaccurate diagnoses of mathematical errors (Sleeman et al., 1989; VanLehn, 1990).

3. As a consequence of the above advantages of attributes, remedial scripts for subskill deficiencies can be prepared beforehand. These scripts may be based on the recommendations of experienced teachers, culled from published studies, or stem from the tutors' "best guesses" about successful remedial strategies. A study using rule space as the basis for remediation has produced positive results (Tatsuoka & Tatsuoka, 1992). Since bugs may be produced capriciously, it is a daunting, if not impossible, task to prescribe remediation.

4. Finally, it is very labor intensive for teachers and researchers to identify, catalog, and diagnose mal-rules [VanLehn (1982) notes that three or four thousand hours were given to hand analyses of protocols]. And even with this expensive input there is no guarantee that all of the possible mal-rules will be found (Sleeman et al., 1989; Payne & Squibb, 1989; VanLehn, 1982). VanLehn (1982, p. 46) noted that even with "excellent tests, an improved DEBUGGY, and a dedicated staff of experienced diagnosticians," 34% of the population of students could not be diagnosed in terms of bugs and slips. VanLehn further noted that the remedial consequences of poor diagnosis for remediation purposes is that the computer system has then, "nothing informative to tell the teacher about the student" (p. 37, original emphasis).

While we are pleased with the within-test stability results for the rule-space analysis, future studies should investigate the stability of the rule-space results over time. In addition, cognitive models for algebra other than the subskill model described here should also be investigated.

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Table 1

Attributes Used in the Q Matrix.

No.	Description
1	Adding a term to both sides of the equation
2	Subtracting a term from both sides of the equation
3	Applying arithmetic order of operations
4	Applying the distributive law
5	Adding or subtracting variable terms
6	Dividing across by the coefficient of x, [resulting in $x=b/a$ when $a=b$]
7	Dividing across by the coefficient of x, [resulting in $x=b/a$ when $a<b$]
8	Dividing across by the coefficient of x, [resulting in $x=b/a$ when $a>b$]
9	Applying symmetry law
10	Evaluating the equation to determine the simplest solution path
11	Applying symmetry law and evaluating the equation to determine the simplest solution path

Table 2

Examples of two Students Bug Patterns for the Nine Parallel Item-Pairs

	Item-Pairs								
Item sets	1	2	3	4	5	6	7	8	9
				Student # 13					
First set	A	B	C	+	B	Ui	D	B	Ui
Second set	A	B	C	+	B	Om	E	B	Cl
				Student # 82					
First set	Cl	F	Ui	+	F	Cl	F	F	F
Second set	Ui	F	C	+	F	G	F	F	Ui

Note.

+ = Correct response
Mal-rules:
A: a + x => ax
B: ax + a => (a + a) x
C: ax = b => x = a/b (when a > b)
D: ax + b + x => (a + b + 1) x
E: ax + b => (a + b) x
F: ax • b = c => ax = c @ b; when • is "+" then @ is "-" and vice versa.
G: ax • bx = cx => a = cx @ bx; when • is "+" then @ is "-" and vice versa.
Other errors:
Cl: Clerical error
Ui: Unidentified
Om: Omitted

Table 3

Frequency of Stable Bugs by Item-Pairs

Bug No.	Item-Pairs 1&25	2&24	3&27	6&23	8&18	10&19	11&20	12&22	13&30
2	8								
3									1
4		1			4			4	
7							2		
9			30						
10			2				1		
14		1							
18							1		
19			10						
20		4							
21		1							
24		1			2		1		
26		2			1		2	1	6
28		3	6	4	3		3	3	
30				2					
32			1				10	1	
33						3			
34	12								10
46						2			
48		1	2						
51		1							
52							1		
59	1	1	1	1	1		1	1	
63							1		
75						1			
98						1			
102	1								2
104							1		
106						1			
116			1	1					
117		1			1		1	1	
121				1					
130			1	1					
131	1								1
No. of different bugs	5	11	9	6	6	5	12	6	5
Frequency	23	17	54	10	12	8	25	11	20

Table 4

Attribute Mastery Patterns for Students 13 and 82.

Attribute	1	2	3	4	5	6	7	8	9	10	11	Knowledge State	D^2
Subset 1	0	0	0	0	0	0	0	0	1	0	1	74	0.0
Subset 2	0	0	0	0	0	0	0	0	1	0	1	74	0.0

Note: The distance, D^2, is the Mahalonobis Distance from the student's point to the centroid of the closest group on the θ and ζ axes.

Appendix A

<u>The Incidence Matrix, Q, for the 18 items, the Item Difficulties and Discrimination Indices, and the Percentage of Mastery for Each Attribute</u>

Items		Attribute 11 12345678901	% Correct	IRT b	IRT a
1	3+x=6+3*2	0110 0000000	74	.71	-1.00
25	4+x=6+2*3	0110 0000000	73	.72	-.94
2	7x+7=14	0100 0100000	81	1.00	1.18
24	12x+12=24	0100 0100000	81	1.12	-1.08
3	16x=4	0000 0001000	63	1.28	-.26
27	28x=7	0000 0001000	54	1.13	.04
6	35=7x	0000 0010100	93	1.20	-2.12
23	24=6x	0000 0010100	92	1.29	-1.90
8	3+6x=18	0100 0010000	77	1.17	-.85
18	8+4x=26	0100 0010000	85	1.30	-1.25
10	4(2x+3)=10x	0101 1010111	83	1.52	-1.05
19	6(x+3)=12x	0101 1010111	81	1.04	-1.13
11	6+4x+x=22	0100 1010000	77	1.38	-.78
20	5+3x+x=16	0100 1010000	76	1.35	-.74
12	98=7+7x	0100 0010100	83	1.39	-1.07
22	75=5+5x	0100 0010100	84	1.55	-1.07
13	x-4=4+2*4	1010 0000000	73	.68	-.98
30	x-6=3+5*3	1010 0000000	67	.74	-.61
% Mastered		6966 9595987 4449 5961597			

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